The Distributional Dynamics of Income and Consumption

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Setting the Scene

- My aim in this lecture is to answer three key questions?
  - How well do consumers insure themselves against adverse shocks?
  - What mechanisms are used?
  - How well does the ‘standard’ incomplete markets model match the data?
- Show how the ***distributional dynamics of wages, earnings, income and consumption*** can be used to uncover the answer to these questions.
  - Draw on many references: Blundell, Pistaferri and Preston, 2008 (BPP) and Blundell, Low and Preston, 2008 (BLP) - http://www.ucl.ac.uk/~uctp39a/

The ***Distributional Dynamics of Income and Consumption*** concern the linked dimensions between wage, income and consumption inequality

- These links between the various types of inequality are mediated by multiple insurance mechanisms, *including:*
- labour supply, taxation, consumption smoothing, informal mechanisms, etc
  - These tie together the underlying elements....
- wages ► earnings ► joint family earnings ► income ► consumption
  - hours
    - family labour supply
      - taxes and transfers
        - self-insurance and partial insurance

‘Insurance’ mechanisms. . .

► These mechanisms will vary in importance across different types of households at different points of their life-cycle and at different points in time.

► The manner and scope for insurance depends on the durability of shocks and access to credit markets

► The objective here is to understand the links between the pattern distributional dynamics of wages, earnings, income and consumption

- **Illustrate with some *key episodes* in the US and UK, also Japan and Australia Figures 1a,..,d.**

Distributional Dynamics of Income, Earnings and Consumption

- Focus on the *Transmission Parameter* or '*Partial Insurance*' approach
  - What do we do?
  - What do we find?
- How well does the *Partial Insurance* approach work?
  - Robustness to alternative representations of the economy
  - Robustness to alternative representations of income dynamics
    - - draw on simulation studies
- Are there other key avenues for 'insurance'?
- What features of the approach need developing/generalising?
  - Start by examining the dynamics of the earnings distribution..

What do we know about the earnings processes facing individuals and families?

Write log income as:

$$y_{i,a,t} = Z'_{i,a,t}\varphi + z_{i,a,t} + B'_{i,a,t}f_i + \varepsilon_{i,a,t} \tag{1}$$

- where Z_{iat} are age, education, interactions etc, z_{iat} is a persistent process of income shocks which adds to the individual-specific trend (by age and time) $B'_{i,a,t}f_i$ and where ε_{iat} is a transitory shock represented by some low order MA process.
- Allow variances (or factor loadings) of z and ε to vary with age, time,..
- For any birth cohort, an useful specification for $B'_{i,t}f_i$ is:

$$B'_{i,t}f_i = p_t\beta_i + \alpha_i \tag{2}$$

Idiosyncratic trends:

▶ The term $p_t\beta_i$ could take a number of forms:

(a) deterministic idiosyncratic trend : $p_t\beta_i = r(t)\beta_i$ where r is known, e.g. $r(t) = t$

(b) stochastic trend in 'ability prices' : $p_t = p_{t-1} + \xi_t$ with $E_{t-1}\xi_t = 0$

▶ Evidence points to some periods of time where each may be of importance (See Blundell, Bonhomme, Meghir and Robin (2008)):

• (a) key component in early working life earnings evolution (Solon et al. using administrative data - see Figure 2). Formally, this is a life-cycle effect. Linear trend looks too restrictive.

• (b) during periods when skill prices are changing across the unobserved ability distribution. Early 1980s in the US and UK, for example. Formally, this is a calender time effect.

A reasonable dynamic representation of income dynamics

- If the transitory shock $\varepsilon_{i,t}$ is represented by a MA(q)

$$v_{it} = \sum_{j=0}^{q} \theta_j \varepsilon_{i,t-j} \text{ with } \theta_0 \equiv 1. \tag{3}$$

- and the permanent shock z_{it} by

$$z_{it} = \rho z_{it-1} + \zeta_{it} \tag{4}$$

With $q = 1$, this implies a 'key' quasi-difference moment restriction

$$\mathrm{cov}(\Delta^\rho y_t, \Delta^\rho y_{t-2}) = \mathrm{var}(\alpha)(1-\rho)^2 + \mathrm{var}(\beta)\Delta^\rho p_t \Delta^\rho p_{t-2} - \rho\theta_1 \mathrm{var}(\varepsilon_{t-2}) \tag{5}$$

where $\Delta^\rho = (1 - \rho L)$ is the quasi-difference operator.

- Note that for large $\rho = 1$ and small θ_1 this implies

$$\mathrm{cov}(\Delta y_t, \Delta y_{t-2}) \simeq \mathrm{var}(\beta)\Delta p_t \Delta p_{t-2}. \tag{6}$$

- Tables 1 & 2 of autocovariances in various panel data on income, Figs 3 & 4:

What do we find?

- importance of age selection (Haider and Solon, AER 2006)

  ► for families, mainly in their 30s,40s and 50s, in the US and the UK the *'permanent-transitory'* model may suffice

- forecastable components and differential trends are most important early in the life-cycle - which limits the importance of learning across the life-cycle

  ► leaves the identification of idiosyncratic trends - $var(\beta)$ - much more fragile

- important to let the variances of the permanent and transitory components vary over time – otherwise strongly reject model

  ► during the late 1970s and early 1980s there were large changes in the variance of permanent and transitory shocks in US and UK (Moffitt and Gottshalk (1994, 2008), Blundell, Low and Preston (2008))

Evolution of the Consumption Distribution

- with self-insurance

▶ Start by assuming at time t each family i maximises the conditional expectation of a time separable, differentiable utility function:

$$\max_C E_t \sum_{j=0}^{T-t} u\left(C_{i,t+j}, Z_{i,t+j}\right)$$

$Z_{i,t+j}$ incorporates taste shifters/non-separabilities and discount rate heterogeneity.

• We set the retirement age at L, assumed known and certain, and the end of the life-cycle at T. We assume that there is no uncertainty about the date of death.

• Individuals can self-insure using a simple credit market with access to a risk free bond with real return r_{t+j}. Consumption and income are linked through the intertemporal budget constraint

$$A_{i,t+j+1} = \left(1 + r_{t+j}\right)\left(A_{i,t+j} + Y_{i,t+j} - C_{i,t+j}\right) \text{ with } A_{i,T} = 0.$$

Consumption Dynamics

- With self-insurance and CRRA preferences

$$u\left(C_{i,t+j}, Z_{i,t+j}\right) \equiv \frac{1}{(1+\delta)^j} \frac{C_{i,t+j}^{\beta} - 1}{\beta} e^{Z'_{i,t+j}\vartheta}$$

  - The first-order conditions become

$$C_{i,t-1}^{\beta-1} = \frac{1+r_{t-1}}{1+\delta} e^{\Delta Z'_{i,t}\vartheta_t} E_{t-1} C_{i,t}^{\beta-1}.$$

- Applying the BLP approximation

$$\Delta \log C_{i,t} \simeq \Delta Z'_{i,t}\vartheta'_t + \eta_{i,t} + \Gamma_{i,t}$$

where $\vartheta'_t = (1-\beta)^{-1}\vartheta_t$, $\eta_{i,t}$ is a consumption shock with $E_{t-1}\eta_{i,t} = 0$, $\Gamma_{i,t}$ captures any slope in the consumption path due to interest rates, impatience or precautionary savings and the error in the approximation is $\mathcal{O}(E_{t-1}\eta_{i,t}^2)$.

- If preferences are CRRA then Γ_{it} does not depend on C_{it}.

Linking the Evolution of Consumption and Income Distributions

- For income we have

$$\Delta \ln Y_{i,t+k} = \zeta_{i,t+k} + \sum_{j=0}^{q} \theta_j \varepsilon_{i,t+k-j}.$$

  - The intertemporal budget constraint is

$$\sum_{k=0}^{T-t} Q_{t+k} C_{i,t+k} = \sum_{k=0}^{L-t} Q_{t+k} Y_{i,t+k} + A_{i,t}$$

where T is death, L is retirement and Q_{t+k} is appropriate discount factor $\prod_{i=1}^{k}(1 + r_{t+i})$, $k = 1, ..., T - t$ (and $Q_t = 1$).

Linking the Evolution of Consumption and Income Distributions

▶ Defining

- $\pi_{i,t} = \sum_{k=0}^{L-t} Q_{t+k} Y_{i,t-k} / (\sum_{k=0}^{L-t} Q_{t+k} Y_{i,t-k} + A_{i,t})$ - the share of future labor income in current human and financial wealth, and
- $\gamma_{t,L} \simeq \frac{r}{1+r}[1 + \sum_{j=1}^{q} \theta_j / (1+r)^j]$ - the annuity factor (for $r_t = r$)
- Show the stochastic individual element $\eta_{i,t}$ in consumption growth is given by

$$\eta_{i,t} \simeq \pi_{i,t} \left[\zeta_{i,t} + \gamma_{t,L} \varepsilon_{i,t} \right]$$

- Accuracy is assessed using simulations in Blundell, Low and Preston (2008).

So **a link between consumption and income dynamics** can be expressed, to order $\mathcal{O}(\|\nu_t\|^2)$, where $\nu_t = (\zeta_t, \varepsilon_t)'$

$$\Delta \ln C_{it} \cong \Gamma_{it} + \Delta Z'_{it}\varphi^c + \pi_{it}\zeta_{it} + \pi_{it}\gamma_{Lt}\varepsilon_{it} + \xi_{it}$$

- Γ_{it} - Impatience, precautionary savings, intertemporal substitution. For CRRA preferences Γ does not depend on C_{t-1}.
- $\Delta Z'_{it}\varphi^c$ - Deterministic preference shifts and labor supply non-separabilities
- $\pi_{it}\zeta_{it}$ - Impact of permanent income shocks - $(1 - \pi_{it})$ reflects the degree to which 'permanent' shocks are insurable in a finite horizon model.
- $\pi_{it}\gamma_{Lt}\varepsilon_{it}$ - Impact of transitory income shocks, $\gamma_{Lt} < 1$ - the annuitisation factor
- ξ_{it} - Impact of shocks to higher income moments,etc

The π parameter

In this model, self-insurance is driven by the parameter π, which corresponds to the ratio of human capital wealth to total wealth (financial + human capital wealth)

$$\pi_{i,t} = \frac{\sum_{k=0}^{L-t} Q_{t+k} Y_{i,t-k}}{\sum_{k=0}^{L-t} Q_{t+k} Y_{i,t-k} + A_{i,t}}$$

- For given level of human capital wealth, past savings imply higher financial wealth today, and hence a lower value of π: Consumption responds less to income shocks (precautionary saving)
  - Individuals approaching retirement have a lower value of π
  - In the certainty-equivalence version of the PIH, $\pi \simeq 1$ and $\alpha \simeq 0$

Partial Insurance

- Under some circumstances, it is possible to insure consumption fully against income shocks. In this case, $\pi = 0$
  - Theoretical problems: Moral hazard, Limited enforcement, etc.
  - Empirical problems: The hypothesis $\pi = 0$ is soundly rejected, references... Attanasio and Davis (1996),....
- Introduce *'partial insurance'* to capture the possibility of 'excess insurance' and also 'excess sensitivity'.

▶ Partial insurance allows some, but not full, additional insurance to persistent shocks. For example, Attanasio and Pavoni (2005) consider an economy with moral hazard and hidden access to a simple credit market. A linear insurance rule can be obtained as an 'exact' solution in a dynamic Mirrlees model with CRRA utility.

Consumption Dynamics with Partial Insurance

Need to generalise to account for additional 'insurance' mechanisms and excess sensitivity - introduce *partial insurance parameters* ϕ_{at} and ψ_{at}

$$\Delta \ln C_{it} \cong \Gamma_{it} + \Delta Z'_{it}\varphi^c + \xi_{it} + \phi_{at}\zeta_{it} + \psi_{at}\varepsilon_{it}$$

- Partial insurance w.r.t. permanent shocks, $0 \leq 1 - \phi_{at} \leq 1$

- Partial insurance w.r.t. transitory shocks, $0 \leq 1 - \psi_{at} \leq 1$

- $1 - \phi_{at}$ and $1 - \psi_{at}$ are the fractions insured and subsume π_{at} and γ_{at} from the self-insurance model.

A Factor Structure for Consumption and Income Dynamics

- We now have a factor structure provides the key panel data moments that link the evolution of distribution of consumption to the evolution of labour income distribution

$$\Delta \ln C_{it} \cong \Gamma_{it} + \Delta Z'_{it}\varphi^c + \phi_{at}\zeta_{it} + \psi_{at}\varepsilon_{it} + \xi_{it}$$

- It describes how consumption updates to income shocks
- It provides key panel data moments that link the evolution of distribution of consumption to the evolution of income
- We compare this with results from a dynamic stochastic simulation of a Bewley economy and other common alternatives.
- Also compare with results under alternative models of the income dynamics.

The key panel data moments

- For log adjusted income:

$$\mathrm{cov}\left(\Delta y_t, \Delta y_{t+s}\right) = \begin{cases} \mathrm{var}\left(\zeta_t\right) + \mathrm{var}\left(\Delta v_t\right) & \text{for } s = 0 \\ \mathrm{cov}\left(\Delta v_t, \Delta v_{t+s}\right) & \text{for } s \neq 0 \end{cases} \tag{7}$$

- Allowing for an MA(q) process, for example, adds $q-1$ extra parameter (the $q-1$ MA coefficients) but also $q-1$ extra moments, so that identification is unaffected.

- For log consumption:

$$\mathrm{cov}\left(\Delta c_t, \Delta c_{t+s}\right) = \phi_t^2 \mathrm{var}\left(\zeta_t\right) + \psi_t^2 \mathrm{var}\left(\varepsilon_t\right) + \mathrm{var}\left(\xi_t\right) \tag{8}$$

for $s = 0$ and zero otherwise.

- For the cross-moments:

$$\mathrm{cov}\left(\Delta c_t, \Delta y_{t+s}\right) = \begin{cases} \phi_t \mathrm{var}\left(\zeta_t\right) + \psi_t \mathrm{var}\left(\varepsilon_t\right) \\ \psi_t \mathrm{cov}\left(\varepsilon_t, \Delta v_{t+s}\right) \end{cases} \tag{9}$$

for $s = 0$, and $s > 0$ respectively.

Identification

▶ The parameters to identify are: $\phi, \psi, \sigma_{\xi}^2, \sigma_{\zeta}^2$, and σ_{ε}^2.

- $E\left(\Delta c_t\left(\Delta y_{t-1}+\Delta y_t+\Delta y_t\right)\right)/E\left(\Delta y_t\left(\Delta y_{t-1}+\Delta y_t+\Delta y_t\right)\right)=\phi$
- $E\left(\Delta c_t \Delta y_{t+1}\right)/E\left(\Delta y_t \Delta y_{t+1}\right)=\psi$
- $E\left(\Delta c_t\left(\Delta c_{t-1}+\Delta c_t+\Delta c_{t+1}\right)\right)-\frac{\left[E(\Delta c_t(\Delta y_{t-1}+\Delta y_t+\Delta y_{t+1}))\right]^2}{E(\Delta y_t(\Delta y_{t-1}+\Delta y_t+\Delta y_{t+1}))}+\frac{\left[E(\Delta c_t \Delta y_{t+1})\right]^2}{E(\Delta y_t \Delta y_{t+1})}=\sigma_{\xi}^2.$
- Note that there is a simple IV interpretation here: ψ is identified by a regression of Δc_t on Δy_t using Δy_{t+1} as an instrument.
- Note again a simple IV interpretation: ϕ is identified by a regression of Δc_t on Δy_t using $(\Delta y_{t-1}+\Delta y_t+\Delta y_{t+1})$ as an instrument.

• BPP show identification with *measurement error in consumption* $c_{i,t}^* = c_{i,t} + u_{i,t}^c$ and *in income* $y_{i,t}^* = y_{i,t} + u_{i,t}^y$. Can show ϕ and $\sigma_{u^c}^2$ are still identified. However, σ_{ε}^2 and $\sigma_{u^y}^2$ cannot be separated.

Non-stationarity

Allowing for non-stationarity and with T years of data

$$E\left(\Delta y_s^*\left(\Delta y_{s-1}^* + \Delta y_s^* + \Delta y_{s+1}^*\right)\right) = \sigma_{\zeta,s}^2$$

for $s = 3, 4, ..., T-1$. The variance of the transitory shock can be identified using:

$$-E\left(\Delta y_s^* \Delta y_{s+1}^*\right) = \sigma_{\varepsilon,s}^2$$

for $s = 2, 3, ..., T-1$. With an MA(1) process for the transitory component:

$$E\left(\Delta y_s^*\left(\Delta y_{s-2}^* + \Delta y_{s-1}^* + \Delta y_s^* + \Delta y_{s+1}^* + \Delta y_{s+2}^*\right)\right) = \sigma_{\zeta,s}^2$$

for $s = 4, 5..., T-2$, and (assuming θ is already identified)

$$-E\left(\Delta y_s^* \Delta y_{s+2}^*\right) = \theta\sigma_{\varepsilon,s}^2$$

for $s = 2, 3, ..., T-2$.

- The other parameters of interest ($\sigma_{u^c}^2, \phi, \psi, \sigma_{\xi}^2$) can also be identified.

Time-varying insurance parameters

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$$\Delta c_s = \xi_s + \phi_s \zeta_s + \psi_s \varepsilon_s + \Delta u_s^c$$

- which would be identified by the moment conditions:

$$\frac{E(\Delta c_s^* \Delta y_{s+1}^*)}{E\left(\Delta y_s^* \Delta y_{s+1}^*\right)} = \psi_s$$
$$\frac{E(\Delta c_s^* (\Delta y_{s-1}^* + \Delta y_s^* + \Delta y_{s+1}^*))}{E\left(\Delta y_s^* \left(\Delta y_{s-1}^* + \Delta y_s^* + \Delta y_{s+1}^*\right)\right)} = \phi_s$$

for all $s = 2, 3, ..., T-1$ and $s = 3, 4, ..., T-2$ respectively.

- These are the moment conditions that we use when we allow the insurance parameters to vary over time.

The BPP Application: US PSID/CEX Data

- “excess smoothness” or “excess insurance” relative to self-insurance

Table 4:

  - College-no college comparison
  - Younger versus older cohorts

Figures 5,6: show implications for variances of permanent and transitory shocks

- Within cohort and education analysis changes the balance between the distribution of permanent and transitory shocks but not the value of the transmission parameters.

Partial Insurance: Wealth

- Excess sensitivity among low wealth households: select (30%) initial low wealth.

Table 5

- Excess sensitivity among low wealth households
  - - use of durables and labour supply among low wealth households?

Alternative Representations

- The complete markets, PIH and autarky cases
- A ‘Bewley’ economy baseline
  - approximation on the distribution of π and positive net-worth constraints
- A simple partial insurance economy
  - all transitory shocks insurable and a component of permanent shocks
- A private information economy
  - with moral hazard and hidden asset accumulation - linear insurance rule as a solution in a Mirrlees model with CRRA utility.
- Advance information
  - known returns from human capital correlated with initial conditions

▶ Robustness Results

- Based on simulating, from the invariant distribution of the economy, an artificial panel of 50,000 households for 71 periods, i.e. a life-cycle - Kaplan and Violante (2008).

▶ **Table 6 baseline, Figure 7a**

▶ **Table 7 know heterogeneous slopes**

▶ **Table 8, Figure 7b degree of persistence in income shocks**

Partial insurance approach performs well and captures many alternative models.

Partial Insurance: Family Transfers and Taxes

Table 9:

- Tax system and transfers provide some insurance to permanent shocks

  ▷ food stamps for low income households studied in Blundell and Pistaferri (2003), 'Income volatility and household consumption: The impact of food assistance programs', special conference issue of JHR,

  ▷ also contains the Meyer and Sullivan paper, 'Measuring the Well-Being of the Poor Using Income and Consumption'

  ▷ little impact of measured family transfers

Family Labour Supply

- Total income Y_t is the sum of two sources, Y_{1t} and $Y_{2t} \equiv W_t h_t$

- Assume the labour supplied by the primary earner to be fixed. Income processes

$$\Delta \ln Y_{1t} = \gamma_{1t} + \Delta u_{1t} + v_{1t}$$

$$\Delta \ln W_t = \gamma_{2t} + \Delta u_{2t} + v_{2t}$$

- Household decisions to be taken to maximise a household utility function

$$\sum_k (1+\delta)^{-k}[U(C_{t+k}) - V(h_{t+k})].$$

$$\Delta \ln C_{t+k} \simeq \sigma_{t+k} \Delta \ln \lambda_{t+k}$$

$$\Delta \ln h_{t+k} \simeq -\rho_{t+k}[\Delta \ln \lambda_{t+k} + \Delta \ln W_{t+k}]$$

with $\sigma_t \equiv U_t'/C_t U_t'' < 0$, $\rho_t \equiv -V_t'/h_t V_t'' > 0$.

- These imply second order panel data moments for $\ln C$, $\ln Y_1$, $\ln Y_2$ and $\ln W$.

▶ Show that the implied moments are sufficient to identify permanent and transitory shock distribution, and their evolution over time, for $\ln Y_1$ and $\ln W$.

▶ When the labour supply elasticity $\rho > 0$ then the secondary worker provides insurance for shocks to Y_1

• **Figure 8**: shows implications for the variance of transitory shocks to household income.

• See also *Attanasio, Berloffa, Blundell and Preston* (2002, EJ), 'From Earnings Inequality to Consumption Inequality', *Attanasio, Sanchez-Marcos and Low* (2005, JEEA), 'Female labor Supply as an Insurance Against Idiosyncratic Risk', and *Heathcote, Storesletten and Violante* (2006), 'Consumption and Labour Supply with Partial Insurance'

Partial Insurance: Durables

- We have seen excess sensitivity among low wealth households: select (30%) initial low wealth.

  also consider

- Impact of durable purchases as a smoothing mechanism?

Table 10

- Excess sensitivity among low wealth households
- For poor households at least - absence of simple credit market
  - Excess sensitivity among low wealth households - even more impressive use of durables among low wealth households: - Browning, and Crossley (2003)

Summary

- The partial insurance approach is ‘robust’ but insurance interpretation sensitive to assumed/estimated persistence in the income series.
  - The incomplete markets model needs modifying to match the data - the transmission parameter is too small relative to the incomplete markets model.
- How well do consumers insure themselves against adverse shocks?
  - 30% of permanent shocks are insured, but
  - Low wealth and low educated
  - imprtant role for tax and welfare
  - Found family labour supply acts as insurance.
  - Durable purchases as insurance to transitory shocks for lower wealth groups.
- Other countries - current circumstances?

What of future research?

◀ Within household insurance: Heathcote, Storesletten and Violante (2006), Lise and Seitz (2005)

◀ Differential persistence across the distribution: optimal welfare results for low wealth/low human capital groups: optimal earned income tax-credits.

◀ Understanding the mechanism and market incentives for excess insurance - Krueger and Perri (2006) and Attanasio and Pavoni (2006).

◀ Advance information, learning and life-cycle income trends - Cuhna, Heckman and Navarro (2005), Guvenen (2006).

◀ Alternative income dynamics e.g. Baker (2001), Haider (2003), Solon (2006)..

◀ The specific use of credit and durables - Browning and Crossley (2007)

◀ The role of housing – see recent disjuncture of the covariance series.....

THE END

Carlos Diaz Alejandro Lecture

What about the evolution of Cross-section Distributions?

For example the distribution of income and consumption in the UK - **Figures 9a,b**

Assuming the cross-sectional covariances of the shocks with previous periods' incomes to be zero, then

$$\begin{aligned}
\Delta\mathrm{Var}(\ln y_t) &= \mathrm{Var}(\zeta_t) + \Delta\mathrm{Var}(\varepsilon_t) \\
\Delta\mathrm{Var}(\ln c_t) &= \pi_t^2\mathrm{Var}(\zeta_t) + \pi_t^2\gamma_t^2\mathrm{Var}(\varepsilon_t) \\
&\qquad + \mathcal{O}(E_{t-1}\|\nu_{it}\|^3) \\
\Delta\mathrm{Cov}(\ln c_t, \ln y_t) &= \pi_t\mathrm{Var}(\zeta_t) + \Delta[\pi_t\gamma_t\mathrm{Var}(\varepsilon_t)] + \mathcal{O}(E_{t-1}\|\nu_{it}\|^3). \qquad (10)
\end{aligned}$$

- Can identify variances of shocks and π
- **Figures 10a,b** show similar structure to US distributions from PSID.
- How well does this work in identifying changes in the variances of the two separate factors? Back to simulated economy - calibrated to UK, **BLP**.

Simulation Experiments

- As before one aim of the Monte Carlo is to explore the accuracy with which the variances can be estimated despite the approximations. In particular, estimates of the permanent variance and of changes in the transitory variance.
- In the base case the subjective discount rate $\delta = 0.02$, also allow δ to take values 0.04 and 0.01. Also a mixed population with half at 0.02 and a quarter each at 0.04 and 0.01.
- In such cases the permanent variance follows a two-state, first-order Markov process with the transition probability between alternative variances, $\sigma^2_{\zeta,L}$ and $\sigma^2_{\zeta,H}$
- For each experiment, BLP simulate consumption, earnings and asset paths for 50,000 individuals. Obtain estimates of the variance for each period from random cross sectional samples of 2000 individuals for each of 20 periods: **Figure 11**

Idiosyncratic Consumption Trends:

Heterogeneous consumption trends Γ_{it}

$$\Delta \ln c_{it} = \eta_{it}^{\varepsilon} + \Gamma_{it} \; + \; \mathcal{O}(E_{t-1}{\eta_{it}}^2)$$

the evolution of variances are modified to give:

$$\Delta \mathrm{Var}\,(\ln y_t) \;\simeq\; \mathrm{Var}(\zeta_t) + \Delta \mathrm{Var}(\varepsilon_t)$$

$$\Delta \mathrm{Cov}(\ln c_t, \ln y_t) \;\simeq\; \pi_t \; \mathrm{Var}(\zeta_t) + \mathrm{Cov}(y_{t-1}, \Gamma_t)$$

$$\Delta \mathrm{Var}(\ln c_t) \;\simeq\; \pi_t^2 \; \mathrm{Var}(\zeta_t) + 2\mathrm{Cov}(c_{t-1}, \Gamma_t)$$

- The evolution of $\mathrm{Var}(\ln c_t)$ is no longer usable since $\mathrm{Cov}(c_{t-1}, \Gamma_t) \neq 0$ for some t.
- The evolution of the cross-section variability in log consumption no longer reflects only the permanent component and so it cannot be used for identifying the variance of the permanent shock. **Figure 12**

Idiosyncratic Income Trends:

The equations for the evolution of the variances become:

$$\Delta \mathrm{Var}(\ln y_t) \simeq \mathrm{Var}(\zeta_t) + \Delta \mathrm{Var}(\varepsilon_t) + 2\mathrm{Cov}(y_{t-1}, f_t)$$

$$\Delta \mathrm{Cov}(\ln c_t, \ln y_t) \simeq \pi_t \, \mathrm{Var}(\zeta_t) + \mathrm{Cov}(c_{t-1}, f_t)$$

$$\Delta \mathrm{Var}(\ln c_t) \simeq \pi_t^2 \, \mathrm{Var}(\zeta_t)$$

where f reflects the idiosyncratic trend

- The evolution of the variance of income is no longer informative about uncertainty.
- The evolution of $\mathrm{Var}\,(\ln c_t)$ can be used to identify the variance of permanent shocks
- The evolution of the transitory variance cannot be identified
- The covariance term is useful only if the levels of consumption are uncorrelated with the income trend, which is unlikely. **Figure 13**

Appendix A: Information and the income process

It may be that the consumer cannot separately identify transitory ε_{it} from permanent ζ_{it} income shocks. For a consumer who simply observed the income innovation ϵ_{it} in $y_{it} = y_{i,t-1} + \epsilon_{it} - \theta_t \epsilon_{i,t-1}$ we have consumption innovation:

$$\eta_{it} = \rho_t(1 - \theta_{t+1})\epsilon_{it} + \frac{r}{1+r}\theta_{t+1}\epsilon_{it} \tag{11}$$

where $\rho_t = 1 - (1+r)^{-(R-t+1)}$. The evolution of θ_t is directly related to the evolution of the variances of the transitory and permanent innovations to income.

- The permanent effects component in this decomposition can be thought of as capturing news about both current and *past* permanent effects since

$$E(\sum_{j=0} \zeta_{i,t-j} | \epsilon_{it}, \epsilon_{i,t-1}, ...) - E(\sum_{j=0} \zeta_{i,t-j} | \epsilon_{i,t-1}, ...) = (1 - \theta_{t+1})\varepsilon_{it}.$$

- This represents the best prediction of the permanent/ transitory split

Appendix B: Linking the Distributions

We begin by calculating the error in approximating the Euler equation.

$$E_t U'(c_{it+1}) = U'(c_{it}) \left(\frac{1+\delta}{1+r} \right) = U'(c_{it} \mathrm{e}^{\Gamma_{it+1}}) \tag{12}$$

for some Γ_{it+1}.

By exact Taylor expansion of period $t+1$ marginal utility in $\ln c_{it+1}$ around $\ln c_{it} + \Gamma_{it+1}$, there exists a $\tilde{c}$ between $c_{it}\mathrm{e}^{\Gamma_{it+1}}$ and c_{it+1} such that

$$\begin{aligned} U'(c_{it+1}) \;=\; & U'(c_{it}\mathrm{e}^{\Gamma_{it+1}}) \Bigg[1 + \frac{1}{\gamma(c_{it}\mathrm{e}^{\Gamma_{it+1}})} [\Delta \ln c_{it+1} - \Gamma_{it+1}] \\ & + \frac{1}{2} \beta(\tilde{c}, c_{it}\mathrm{e}^{\Gamma_{it+1}}) [\Delta \ln c_{it+1} - \Gamma_{it+1}]^2 \Bigg] \end{aligned} \tag{13}$$

where $\gamma(c) \equiv U'(c)/cU''(c) < 0$ and $\beta(\tilde{c}, c) \equiv \left[\tilde{c}^2 U'''(\tilde{c}) + \tilde{c} U''(\tilde{c}) \right] / U'(c)$.

Taking expectations

$$E_t U'(c_{it+1}) \;=\; U'(c_{it}\mathrm{e}^{\Gamma_{it+1}}) \left[1 + \frac{1}{\gamma(c_{it}\mathrm{e}^{\Gamma_{it+1}})} E_t[\Delta \ln c_{it+1} - \Gamma_{it+1}] \right.$$
$$\left. + \frac{1}{2} E_t \left\{ \beta(\tilde{c}, c_{it}\mathrm{e}^{\Gamma_{it+1}}) [\Delta \ln c_{it+1} - \Gamma_{it+1}]^2 \right\} \right] \tag{14}$$

Substituting for $E_t U'(c_{it+1})$ from (12),

$$\frac{1}{\gamma(c_{it}\mathrm{e}^{\Gamma_{it+1}})} E_t[\Delta \ln c_{it+1} - \Gamma_{it+1}] + \frac{1}{2} E_t \left\{ \beta(\tilde{c}, c_{it}\mathrm{e}^{\Gamma_{it+1}}) [\Delta \ln c_{it+1} - \Gamma_{it+1}]^2 \right\} = 0$$

and thus

$$\Delta \ln c_{it+1} = \Gamma_{it+1} - \frac{\gamma(c_{it}\mathrm{e}^{\Gamma_{it+1}})}{2} E_t \left\{ \beta(\tilde{c}, c_{it}\mathrm{e}^{\Gamma_{it+1}}) [\Delta \ln c_{it+1}\mathrm{e}^{\Gamma_{it+1}}]^2 \right\} + \varepsilon_{it+1} \tag{15}$$

where the consumption innovation ε_{it+1} satisfies $E_t \varepsilon_{it+1} = 0$. As $E_t \varepsilon_{it+1}^2 \to 0$, $\beta(\tilde{c}, c_{it}\mathrm{e}^{\Gamma_{it+1}})$ tends to a constant and therefore by Slutsky's theorem

$$\Delta \ln c_{it+1} = \varepsilon_{it+1} + \Gamma_{it+1} \;+\; \mathcal{O}(E_t|\varepsilon_{it+1}|^2). \tag{16}$$

If preferences are CRRA then Γ_{it+1} does not depend on c_{it} and is common to

all households, say Γ_{t+1}. The log of consumption therefore follows a martingale process with common drift

$$\Delta \ln c_{it+1} = \varepsilon_{it+1} + \Gamma_{t+1} \; + \; \mathcal{O}(E_t|\varepsilon_{it+1}|^2). \tag{17}$$

The second step in the approximation is relating income risk to consumption variability. In order to make this link between the consumption innovation ε_{it+1} and the permanent and transitory shocks to the income process, we loglinearise the intertemporal budget constraint using a general Taylor series approximation, see Blundell, Low and Preston (2005).

Appendix C: Simulating the variance of permanent shocks

- Transitory shocks are assumed to be $i.i.d.$ within period with variance growing at a deterministic rate.
- The permanent shocks are subject to stochastic volatility.
- The permanent variance as following a two-state, first-order Markov process with the transition probability between alternative variances, $\sigma^2_{v,L}$ and $\sigma^2_{v,H}$, given by β.

$$\begin{array}{c|cc} & \sigma^2_{v,L} & \sigma^2_{v,H} \\ \hline \sigma^2_{v,L} & 1-\beta & \beta \\ \sigma^2_{v,H} & \beta & 1-\beta \end{array} \tag{18}$$

- Consumers believe that the permanent variance has an ex-ante probability β of changing in each t. In the simulations, the variance actually switches only once and this happens in period S, which we assume is common across all individuals.

How well does the standard incomplete markets model account

for the observed distributional dynamics?

► Permanent - transitory model of earnings alone cannot explain the joint distributional dynamics of consumption and earnings

• BPP (2008) suggest some additional partial insurance mechanism: some part of the permanent shock is insured.

• Guvenen (2006) and this paper (G-S) suggest idiosyncratic trends: permanent-transitory specification over-estimates the role of permanent unanticipated effects, but have to include learning.

• Kaplan and Violante (2008) show that a (very) little less persistence than in the permanent-transitory model can do a much better job in matching the distributions but then find too little insurance later in the life-cycle.

Summary so far....

◀ The aim: to analyse the transmission from income to consumption inequality

◀ Specifically to examine the disjuncture in the evolution of income and consumption inequality in the US & UK in the 1980s - argue that a key driving force is the nature and the durability of shocks to labour market earnings

- a dramatic change in the mix of permanent and transitory income shocks over this period - revisionists?
- the growth in the persistent factor during the early 1980s inequality growth episode carries through into consumption

◀ But the transmission parameter is too small relative to the standard incomplete markets model. Even more so, $\pi = 1$ is a bad approximation.

- about 30% of permanent shocks are insured (but not for the low wealth).

What next?

- Robustness to assumptions about the nature of the economy and the nature of the shocks
  - Credit market and insurance assumptions
  - Persistence of ‘shocks’ and advance information
  - Simulation studies for panel data and cross-section distributions under alternative assumptions
- Additional ‘Insurance’ Mechanisms?
  - Family Transfers, taxes and welfare
  - Individual and family labor supply
  - Durable replacement

Anticipation

Test $cov(\Delta y_{t+1}, \Delta c_t) = 0$ for all t, p-value 0.3305

Test cov(Δy_{t+2}, Δc_t) = 0 for all t, p-value 0.6058

Test cov(Δy_{t+3}, Δc_t) = 0 for all t, p-value 0.8247

Test cov(Δy_{t+4},Δc_t) = 0 for all t, p-value 0.7752

◀ We find little evidence of anticipation.

◀ This 'suggests' the shocks that were experienced in the 1980s were largely unanticipated.

◀ These were largely changes in the returns to skills, shifts in government transfers and the shift of insurance from firms to workers.

A Bewley economy

▶ Simulate a life-cycle version of the standard incomplete markets model e.g. Huggett (1993). (Kaplan and Violante (2008)).

- Markets are incomplete: the only asset available is a single risk-free bond.
- Households have time-separable expected CRRA utility

$$E_0 \sum_{t=1}^{T} \beta^{t-1} m_t u(C_{it})$$

- Households enter the labor market at age 25, retire at age 60 and die at age 100.
- Assume survival rate $m_t = 1$ for the first T^{work} periods, so that there is no chance of dying before retirement.
- Discount factor: $.964$ with interest rate to match an aggregate wealth-income ratio of 3.5.

▶ **Income process**:

- Stochastic after-tax income, Y_{it}: deterministic experience profile, a permanent and transitory component; initial permanent shock is drawn from normal distribution.
- deterministic age profile for income from PSID data, peaks after 21 years at twice the initial value and then declines to about 80% of peak.
- variance of permanent shocks 0:02; variance of transitory shocks 0:05; as in BPP.
- The initial variance is set at 0:15 to match the dispersion at age 25.
- Households begin their life with initial wealth A_{i0}, face a lower bound on assets $\underline{A}$.
- Treat income Y_{it} as net household income after all transfers and taxes, also consider taxes on labor income through a non-linear tax rule $\tau(Y_{it})$ reflecting the redistribution in the US tax system.
- Similar for 'cross-section' simulations for UK comparison.

- **Advance information I**
  - a proportion of the shocks are known in advance to the consumer
  - the permanent change in income at time t consists of two orthogonal components, one that becomes known to the agent at time t, the other is in the agent's information set already at time $t-1$.
- **Advance information II:**
  - the income process includes heterogeneous slopes in individual income profiles:

$$y_{it} = f_{1i}t + y_{it}^{P} + \varepsilon_{it}$$

with $E(f_{1i}) = 0$, in the cross-section and $var(f_{1i}) = \sigma_{f_1}$, assume that f_{1i} is learned by the agents at age zero.

- **Table XIa,b: advance information; Table XII: persistence of shocks**

Additional ‘Insurance’ Mechanisms

- Redistributive mechanisms: social insurance, transfers, progressive taxation
  - Gruber; Gruber and Yelowitz; Blundell and Pistaferri; Kniesner and Ziliak
- Family and interpersonal networks
  - Kotlikoff and Spivak; Attanasio and Rios-Rull
- Family Labour Supply: Wages► earnings► joint earnings► income ...
  - Stephens; Heathcote, Storesletten and Violante; Attanasio, Low and Sanchez-Marcos
- Durable replacement
  - Browning and Crossley

▶ **The key panel data moments** become:

$$
\begin{aligned}
Var(\Delta c_t) &\simeq \beta^2\sigma^2 s^2 Var(v_{1t}) + \beta^2\sigma^2(1-\rho)^2(1-s)^2 Var(v_{2t}) \\
&\quad +2\beta^2\sigma^2(1-\rho)s(1-s)Cov(v_{1t}, v_{2t}) \\
Var(\Delta y_{1t}) &\simeq Var(v_{1t}) + \Delta Var(u_{1t}) \\
Var(\Delta y_{2t}) &\simeq (1-\psi)^2 Var(u_{2t}) - \beta^2\rho^2 s^2 Var(v_{1t}) \\
&\quad +\beta^2\sigma^2(1-\rho)^2 Var(v_{2t}) - 2\beta^2\sigma(1-\rho)sCov(v_{1t}, v_{2t}) \\
Var(\Delta w_t) &\simeq Var(v_{2t}) + \Delta Var(u_{2t})
\end{aligned}
$$

where

- $\beta = 1/(\sigma + \rho(1-s))$.
- s_t is the ratio of the mean value of the primary earner's earnings to that of the household $\overline{Y}_{1t}/\overline{Y}_t$